

On the workload and distribution of grades in the Keller individualized teaching

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Courses based on the Keller method (also named Personalized System of Instruction - PSI) present several interesting features and results, such as final grade distributions which are upside down (the majority of students achieving the highest grades) and a workload for the elaboration and implementation considerably greater than that involved in traditional courses. In the present paper, we propose a mathematical model to describe the scheme of a course implemented according to the Keller plan. This model predicts the time evolution of the distribution of students per unit of content, predicts the upside down effect in the final grades and establishes conditions under which this effect can be observed. The model also provides a quantification of the workload spent in implementing assessments, so it can be an useful tool for those planning or interested in further investigations on Keller courses.

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I. INTRODUCTION

In his paper of 1968 [1], Keller describes his instructional plan (also named Personalized System of Instruction - PSI) as one through which the student can move (at his own pace), showing mastery, not being forced to go ahead until he is ready. Together with Bloom's learning for mastery, the PSI belongs to a class of approaches usually named as "mastery learning", which has shown "positive effects on the examination performance of students in colleges, high schools, and the upper grades in elementary schools" [2]. The mastery learning emerged as an alternative to "traditional courses", here defined as those where students spend most of their time in classroom attending lectures, and tests are only meant to score the success obtained at the course.

The PSI originally started as a result of the application of the reinforcement theory, according to which the instruction process must be based on presentation, performance and consequences, maximizing the frequency of reinforcement and reducing the aversive consequences of errors [3]. This led to the main PSI features, among them: mastery, self-pacing, linear small-step sequenced materials (each one related to a unit of content), repeated testing and immediate feedback [4].

In a Keller course, assessments are not only a measure of success, as usually done in the mentioned traditional scheme, but also formative. An assessment is considered to be formative if it "provides students opportunities to revise and hence improve the quality of their thinking and learning" [5]. In the Keller method, assessments can be viewed as formative, as in this method the student who does not achieve mastery of a unit can take other versions

of that unit assessment, being this a procedure viewed by the students as one that helps the learning process [6]. The formative aspect of these assessments is enhanced by the systematic feedback received by the students right after these assessments [1, 4]. Both formative assessments and systematic feedback are presently considered fundamental to learning [5].

Another interesting feature of a Keller course is that students spend most of their time in classroom doing assessments, receiving feedback or studying the instructional material: "instead of responding passively to a lecture, students must actively read, study, and respond in writing to questions over textual materials" [7]. In PSI, lectures are "...vehicles of motivation, rather than sources of critical information" [1] and are typically short [7]. This drastic change in the role of lectures in classroom can also be justified by researches showing low retention of information in the traditional science lectures, which can be a consequence of the limited capacity of our working memory [8].

On the mastery issue, in traditional lecture courses, some researches have shown that students master a maximum of 20-30% of the new key concepts presented [8–10]. On the other hand, another remarkable change brought by the Keller scheme (and other mastery learning strategies) was the requesting of mastery in a given unit of content, in the PSI case letting "...the student go ahead to new material only after demonstrating mastery of that which preceded, before to go to the next one" [1], which is related to another interesting feature of the Keller method: the "production of a grade distribution that is upside down" [1], which means that, instead of a typical bell-shaped curve for the final grades found in traditional courses [10, 11], the PSI generates "exponential shaped" curves.

Keller's method has been applied in many courses and the recent advances in the technology of information are being considered for implementing and improving the

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Keller scheme [12, 13]. For instance, “...characteristics of Keller’s personalized system of instruction are key elements of Khan Academy’s learning philosophy” [14]. Specifically, the description of several applications of the Keller scheme in Physics courses can be found in the literature (see, for instance, Refs. [15]). It can be also found in literature reports that Keller’s method generally produces superior student achievement when compared with the traditional scheme [16–18]. Kulik et al [16], based on a meta-analysis of 72 studies, reveals not only that students in a PSI course outperformed traditional courses in measures of student satisfaction, but also reveals that PSI students actually get better grades [18].

Although evidences derived from observations indicate success of the Keller scheme in many aspects, it began to decline in popularity during the 1980’s. Among others, the high workload [12, 19] associated with the implementation of a Keller course is one of the possible reasons for this decline: “Training and supervising proctors along with developing the course materials and grading multiple test attempts from each student was an onerous process and many gave this up in favor of more traditional methods” [12].

In the present paper, we present a mathematical model to describe a course done according to the Keller plan. With this tool, we investigate the time evolution of the distribution of students per unit of content, the situations where we can observe the upside down effect in the grades, the workload spent with each assessment and the total workload with assessments.

The paper is organized as follows. In Sec. II, we discuss a mathematical model to describe a very general PSI course. In Sec. III, we propose some approximations which allow a visualization of several interesting aspects of PSI courses. In Sec. IV we make our final considerations.

II. A GENERAL EXACT MODEL

In traditional courses, concepts and materials are divided into N_u units of content, which “...correspond, in many cases, to chapters in the textbook used in teaching” [10]. Usually, N_a (being $N_a \geq N_u$) tests are administered to the students. Usually each test covers the content of a given unit, so we can say that in traditional courses $N_a \approx N_u$. To the teacher, a test is “...an evaluation device that determines who learned those concepts well and who did not” [10]. To the students, each test means “...the end of instruction on the unit and the end of the time they need to spend working on those concepts” and most of the time “...the only chance to demonstrate what they learned” [10]. “After the test is administered and scored, marks are recorded in a grade book, and instruction begins on the next unit, where the process is repeated” [10]. The structure of a course according to the Keller plan [1] is very different and can be modeled as follows.

The course content is divided into N_u units, organized in a definite numerical order $(1, \dots, N_u)$, and the students have to show mastery in each unit by passing assessments (tests, works, etc.). Let us consider that there are N_a total opportunities of such assessments related to these N_u units, so that, if the student fails to pass an assessment on the first opportunity, there can be other opportunities to be used. It can be noticed that in typical Keller courses $N_a \gg N_u$, whereas in traditional courses $N_a \approx N_u$. In addition, in a Keller course it can be applied N_e final examinations (but commonly $N_e = 1$), each one of them applied at the same time for all students. Usually, a certain percentage λ_u of the course grade is based on the number of units successfully completed during the term, a percentage λ_f is based on the final examinations, whereas the percentage $1 - (\lambda_u + \lambda_f)$ is associated with exercises (laboratory works, etc.). This means that there can have a minimal number N_c of units of content to be successfully completed by the students to get approval.

Let us consider a course based on the Keller plan, with N_s students. In Fig. 1 we represent in a Cartesian plane the scheme of assessments of a Keller course, with $N_u = 9$, $N_c = 5$ and $N_a = 14$. In the horizontal axis we set a number for each opportunity of assessment, whereas in the vertical axis we set a number for each unit of content. In this manner, the point $(1, 1)$ means that in the first opportunity of assessment the students can be trying to pass the assessment related to the first unit of content. The point $(2, 1)$ means that in the second opportunity of assessment, students are trying to pass the assessment related to the first unit of content, whereas the point $(2, 2)$ shows that in the same opportunity of assessment there are students trying to pass the assessment related to the second unit of content. The other points have analogous meaning. Considering $N_{(i,j)}$ as the number of students in the i th opportunity of assessment, trying to pass the assessment related to the j th unit of content, one can establish relations to describe the propagation of the number of students from a point to another:

$$N_{(i+1,1)} = \alpha_{(i,1)} N_{(i,1)} \quad (i \geq 1), \quad (1)$$

$$N_{(i,i)} = \beta_{(i-1,i-1)} N_{(i-1,i-1)} \quad (i > 1), \quad (2)$$

$$N_{(i+1,j)} = \alpha_{(i,j)} N_{(i,j)} + \beta_{(i,j-1)} N_{(i,j-1)} \quad (i \geq j > 1), \quad (3)$$

where i and $j = 1, 2, \dots$, the α coefficients are related to the failure in passing an assessment, whereas the β coefficients are related to the success (mastery in a given unit of content).

In Fig. 1, the horizontal (red) arrows represent the propagation - mediated by the α coefficients - of the number of students that failed a given assessment, whereas the diagonal (blue) arrows represent the propagation - mediated by the β coefficients - of the number of students that obtain success. The points indicated by dark circles mean possible real situations of assessments, whereas the

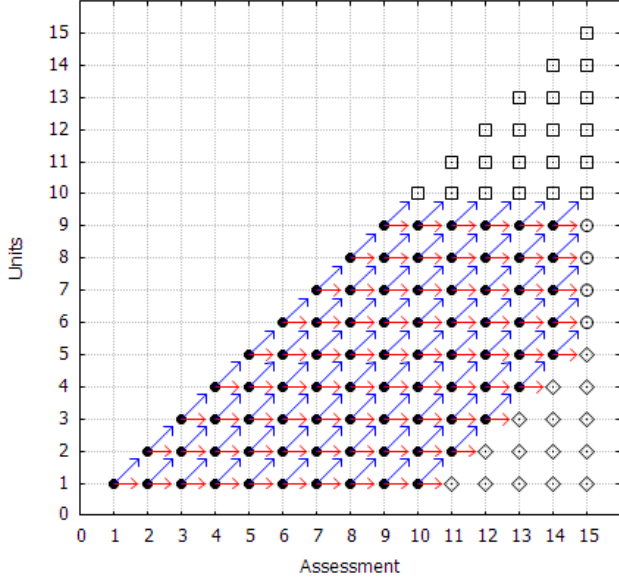


FIG. 1. The scheme of assessments of a Keller course with $N_u = 9$, $N_c = 5$ and $N_a = 14$. In the horizontal axis we set a number for each opportunity of assessment, whereas in the vertical axis we set a number for each unit of content. The points indicated by dark circles mean possible real situations of assessment. The points indicated by squares, diamonds and circles mean virtual situations.

points indicated by squares, diamonds and circles are virtual points, which means that there are no real assessment related to these points. These virtual points are inserted because they will be useful (see Sec. III) in the analysis of the time evolution of the number of students in each unit: those who have already concluded all real N_u units will “occupy” virtual units (squares); those students that have not concluded the minimal number N_c of units will occupy the diamond points; and those who concluded between N_c and N_u units occupy the circle points. This can be illustrated through the following examples: (i) students at the point (14, 9) who pass this assessment complete all units, having no more assessments to pass, reach the “virtual” point (15, 10); (ii) those at the point (14, 9) and that do not pass this assessment go to the “virtual” point (15, 9); (iii) and those at the point (14, 5) and that pass this assessment reach the “virtual” point (15, 6).

According to Ref. [1], in a Keller course, the instructor has the following responsibilities: (a) “the selection of all study material used in the course”; (b) “the organization and the mode of presentation of this material”; (c) “the construction of tests and examinations”; (d) “the final evaluation of each student’s progress”; (e) “provide lectures, demonstrations and discussion opportunities...”. The proctors evaluate readiness tests as satisfactory or unsatisfactory [1]. Here we are interested in estimating the workload of a Keller plan with assessments, so that we are interested in the time spent on the activities (c)

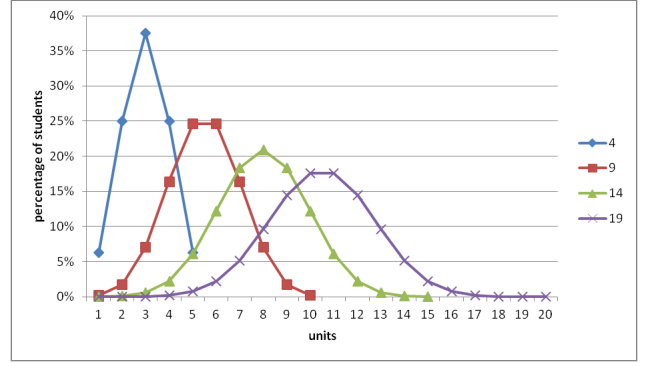


FIG. 2. For the case $(\alpha, \beta) = (1/2, 1/2)$, the horizontal axis gives the number of each unit of content, whereas the vertical axis exhibits the percentage of students ($N_{(i,j)}/N_s$) who are able to do the assessment correspondent to each unit. After 4 assessments, the situation is described by the blue line (diamonds), which shows $N_{(5,j)}/N_s$ for $j = 1..5$; after the 9th assessment, the red line (squares) shows $N_{(10,j)}/N_s$ for $j = 1..10$; after the 14th assessment, $N_{(15,j)}/N_s$ for $j = 1..15$ is described by the green line (triangles); and after the 19th assessment, the situation is described by $N_{(20,j)}/N_s$ for $j = 1..20$, which is indicated by the purple line (crosses).

and (d), and also with the time spent by the proctors.

The total workload W with assessments of a Keller course are written as

$$W = U + V, \quad (4)$$

where U is the workload related with the assessments of the units of content, whereas V is the workload related with the final examinations. The workloads U and V can be given by

$$U = \sum_{i=1}^{N_a} U_i, \quad V = \sum_{i=1}^{N_e} V_i, \quad (5)$$

where U_i is the workload related with the assessments in the i th opportunity, and V_i is the workload related with the i th examination. Let us consider the following values: $\tilde{\tau}_{(i,j)}$ is the time spent with the elaboration of the assessment related to the unit j , performed in the i th opportunity of assessment; $\tau_{(i,j;k)}$ is the time spent with the correction of the assessment (and feedback) related to k th student performing the j th unit in the opportunity of assessment number i ; $\tilde{t}_{(i)}$ is the time spent with the elaboration of the i th final examination; $t_{(i;k)}$ is the time spent with the correction (and feedback) related with the i th final examination related to k th student.

We have

$$U_i = \sum_{j=1}^{\mathcal{F}(i)} \left[\left(\sum_{k=1}^{N_s} \tau_{(i,j;k)} \right) + \tilde{\tau}_{(i,j)} \right], \quad (6)$$

where $\mathcal{F}(i) = i$, if $i \leq N_u$, and $\mathcal{F}(i) = N_u$, if $i > N_u$.

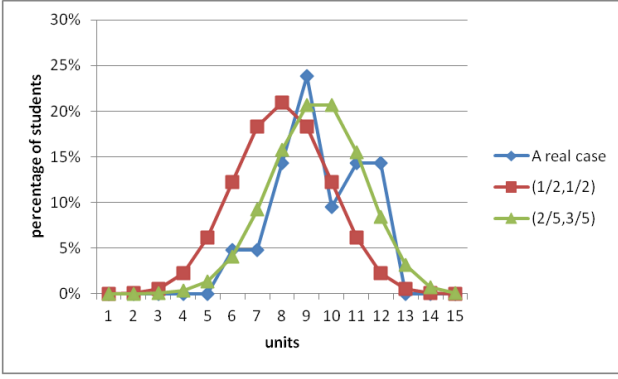


FIG. 3. The horizontal axis shows the number of each unit of content, whereas the vertical axis exhibits the percentage of students ($N_{(i,j)}/N_s$) who are able to do the assessment correspondent to each unit, after the 14th assessment, for the cases: $(\alpha, \beta) = (1/2, 1/2)$ (squares in the red line); $(\alpha, \beta) = (2/5, 3/5)$ (triangles in the green line); and a real case described in Ref. [6] (diamonds in the blue line).

The workload with the i th final examination is given by

$$V_i = \left(\sum_{k=1}^{N_s} t_{(i,k)} \right) + \tilde{t}_{(i)}. \quad (7)$$

Using Eqs. (6) and (7) in (4), we get an exact formula for the total workload with assessments, accounting for a very general possible Keller course and variations of it.

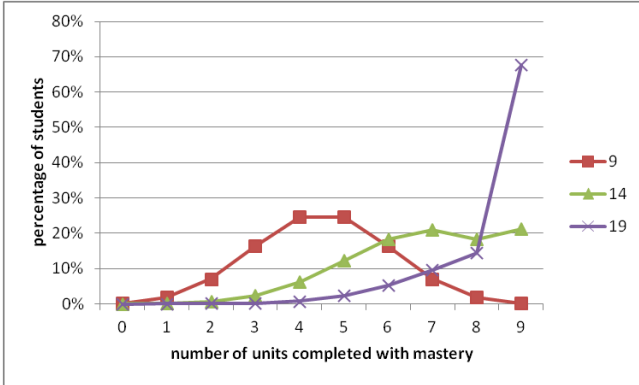


FIG. 4. The horizontal axis shows the number of units completed with mastery. The vertical axis exhibits the percentage of students. The cases considered are defined by $(\alpha, \beta) = (1/2, 1/2)$, $N_u = 9$ and for the following values of N_a : for $N_a = 9$ (squares in the red line); for $N_a = 14$ (triangles in the green line); for $N_a = 19$ (crosses in the purple line).

III. APPROXIMATE MODELS AND APPLICATIONS

From now on, let us focus on having an approximate mean behavior of a course based on the Keller plan.

Then, we introduce certain approximations (labeled with Roman numerals), with the intention of generating a more treatable model for a general description for this system. Our approximation considers that: (i) there is no dropping out of the course, so that the number N_s of students at any time will be always the same; (ii) all students use all the available opportunities of assessments; (iii) in each opportunity, a given student try to pass in just one of the N_u units of content; (iv) $\tilde{\tau}_{(i,j)} \approx \tilde{\tau}$; (v) $\tau_{(i,j;k)} \approx \tau$; (vi) $\tilde{t}_{(i)} \approx \tilde{t}$. Considering these approximations, we can write:

$$\alpha_{(i,j)} + \beta_{(i,j)} = 1, \quad (8)$$

$$\sum_{k=1}^{N_s} \tau_{(i,j;k)} \approx N_{(i,j)} \tau, \quad (9)$$

$$\tilde{\tau}_{(i,j)} \approx \theta(N_{(i,j)}) \tilde{\tau}, \quad (10)$$

where $N_{(i,j)}$ is the number of students at the position (i, j) , and $\theta(N_{(i,j)}) = 1$ if $N_{(i,j)} > 0$ and zero otherwise. The θ step function is used here to indicate that the time spent with the elaboration of an assessment (for a given unit) only have to be considered if there is a number of students different from zero at position (i, j) .

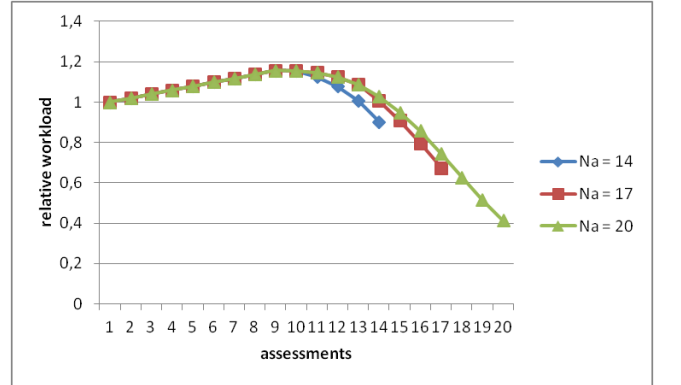


FIG. 5. The relative workload U_i/U_1 (vertical axis) versus number of assessments (horizontal axis), for the case $(\alpha, \beta) = (1/2, 1/2)$, $N_u = 9$ and $N_c = 5$: for $N_a = 14$ (diamonds in the blue line); for $N_a = 17$ (squares in the red line); for $N_a = 20$ (triangles in the green line).

Hereafter, we will consider the approximate model where

$$\alpha_{(i,j)} = \alpha, \quad \beta_{(i,j)} = \beta, \quad (11)$$

with α and β constants. Using this in Eqs. (1), (2) and (3), we get $N_{(2,1)} = \alpha N_s$, $N_{(2,2)} = \beta N_s$, $N_{(3,1)} = \alpha^2 N_s$, $N_{(3,2)} = 2\alpha\beta N_s$, $N_{(3,3)} = \beta^2 N_s$ and so on, so that the coefficients for $N_{(2,j)}$ are related to $(\alpha + \beta)^{2-1}$, and those for $N_{(3,j)}$ are related to $(\alpha + \beta)^{3-1}$, and for $N_{(i,j)}$ are related to $(\alpha + \beta)^{i-1}$. Using the binomial coefficients we obtain

$$N_{(i,j)} = \frac{(i-1)! \alpha^{i-j} \beta^{j-1}}{(j-1)! (i-j)!} N_s. \quad (12)$$

For $(\alpha, \beta) = (1/2, 1/2)$, we get the time (given in terms of the number of assessments already done) evolution of the class as shown in Fig. 2, which exhibits the situations of the class after the 4th, 9th, 14th and 19th assessments. For instance, after 4 assessments (blue line with points indicated by diamonds), the situation is described by $N_{(5,j)}/N_s$, and the figure shows that $N_{(5,1)}/N_s \approx 6\%$ of the students need to redo the assessment of the 1st unit, whereas $N_{(5,2)}/N_s \approx 25\%$ are able to do the assessment of the 2nd unit, $N_{(5,3)}/N_s \approx 38\%$ are able to do the assessment of the 3rd unit, $N_{(5,4)}/N_s \approx 25\%$ of the 4th unit, and $N_{(5,5)}/N_s \approx 6\%$ are able to do the assessment of the 5th unit. The red line (with points indicated by squares) shows that after the 9th assessment and for $N_u = 9$, among other results, $N_{(10,9)}/N_s \approx 1.8\%$ of the students are able to do the assessment of the last 9th unit and $N_{(10,9)}/N_s \approx 0.2\%$ are at the “virtual” unit 10, which means that these students have already shown mastery in all 9 units. The purple line (with points indicated by crosses) shows, after the 19th assessment and for $N_u = 9$, among other results, that the sum over the virtual units $\sum_{j=10}^{20} N_{(20,j)}/N_s \approx 67.6\%$ gives the part of the students that have already shown mastery in all the $N_u = 9$ units of content. The lines blue (diamonds), red (squares), green (triangles) and purple (crosses) exhibit the behavior of a “wave packet” propagating from the left to right in Fig. 2. For other values of α and β , we get the same behavior, but as β increases (it becomes easier to pass assessments) the peak of the wave packet propagates faster, showing that students obtain a faster progress (mastery) in the units of content.

In Fig. 3, we exhibit the wave packet behavior (diamonds in the blue line) found in a real situation (discarding waivers) of a course based on the Keller method and described in Ref. [6]. In the same figure, we also show the formation of wave packets for $(\alpha, \beta) = (2/5, 3/5)$ (triangles in the green line) and $(\alpha, \beta) = (1/2, 1/2)$ (squares in the red line), reinforcing that the approximation adopted here, despite its limitations, is able to describe the essential feature of the time evolution of the distribution of students per unit of content in the Keller scheme.

To investigate the production in Keller courses of grade distributions that is upside down [1], let us consider the hypothesis that as the number of units of content in which a student shows mastery increases, the probability of achieving the best final grades also increases. Then, we assume that the behavior of final grades is intrinsically related to the mastery level reached by the students. Let us consider, for example, the model described by Eq. (12) with $(\alpha, \beta) = (1/2, 1/2)$, and also $N_u = 9$ for the following situations: when $N_a = 9$, $N_a = 14$ and $N_a = 19$. In Fig. 4 we observe the distribution of students versus the maximum number of the units of content for which they obtained mastery at the end of the course. For instance, the squares in the red line show that for $N_a = 9$ we have a distribution of mastery in PSI which resembles a bell-shaped curve, which is typical of traditional courses [10]. We interpret this approximation between

the results for mastery in PSI and traditional courses as a consequence of our choice of a theoretical model of a Keller scheme with $N_a = N_u$, being the relation $N_a \approx N_u$ typical in traditional courses [10]. However, in real PSI courses $N_a > N_u$ and one can observe in Fig. 4 that the distribution of mastery suffers an inversion as N_a becomes greater than N_u . For $N_a = 14$ (triangles in the green line), we already observe a deformation of the “bell-shaped” curve. For $N_a = 19$ (crosses in the purple line), we finally observe an inversion: a transition from a bell-shaped to an “exponential-shaped” curve, indicating that the majority of students obtained mastery in all $N_u = 9$ units. Then, assuming that the behavior of final grades is related to the mastery level reached by the students, an inversion of the mastery curve can be directly mapped into an inversion of final grades, which corresponds to the upside down effect mentioned by Keller [1, 11]. But, we remark that the present model reveals that this inversion does not occurs for any value of N_a , but as the number N_a enhances in comparison to N_u , also enhances the possibility of appearance of the mentioned inversion effect.

In order to quantify the workload with each assessment of the PSI, relative to the workload of each assessment of the traditional scheme, we begin by emphasizing that the value of U_1 (see Eq. (5)) corresponds to the workload of the assessment in the 1st opportunity when, in our approximate model, all students are doing an assessment for a same unit of content. In the PSI this 1st assessment is the most similar to those found in traditional schemes. Then, considering that the mean workload per assessment in the traditional scheme (labeled as U_{trad}) is

$$U_{trad} \approx U_1, \quad (13)$$

this value serves as a basis for the building of the relative workload $U_i/U_{trad} \approx U_i/U_1$. Considering the additional approximation

$$\tau = \tilde{\tau}, \quad (14)$$

we can write $U_1 = (N_{(1,1)} + 1)\tau \approx U_{trad}$, and obtain the results shown in Fig. 5, which shows that the relative workload increases, reaching its maximum point at the N_u th assessment, and then starts to decrease, becoming even smaller than U_{trad} .

The minimum total workload with assessments in a Keller course, in this approximate model, will occur when all students pass each assessment, so that $N_{(i,j)} = \delta_{ij}N_s$, for $i \leq N_u$ and zero otherwise (δ is the Kronecker delta). The maximum total workload will occur when each student uses the complete set of N_a assessments available or, in other words, N_s is completely distributed in the points indicated by dark circles in Fig. 1. In Fig. 6, considering $N_u = 9$, $N_c = 5$ and $N_e = 1$, it is shown the relative total workload W/U_1 for some cases. For instance, for $N_a = 20$ and $(\alpha, \beta) = (3/5, 2/5)$, we have $W \approx 22 \times U_1$. If we consider a typical traditional scheme with a workload

$$W_{trad} = 4 \times U_{trad}, \quad (15)$$

then we have $W \approx 5.5 \times W_{trad}$. Considering the maximum and minimum values shown in Fig. 6, we have, for this case, $2.5 \leq W/W_{trad} \leq 5.8$, which shows that our model is in agreement with Eyre [12] and Silberman [19] when it points to a considerable more workload in implementing assessments in the PSI in comparison to traditional courses.

Although in the last paragraph above we have shown numbers for the ratio W/W_{trad} , these values are constrained by the approximations, specifically Eqs. (14) and (15). Just replacing the approximation (13) by $U_{trad} \approx \gamma U_1$, where the factor $\gamma < 1$ can be introduced taking into account that the time τ which appears in U_1 is related to correction of an assessment of a given student and give him the correspondent feedback, what in the original PSI scheme [1] is done by tutors (proctors). If we compute in τ , in addition to the time spent by the proctor with these activities, the time spent by the instructor to find, train and supervise tutors, we can find the presence of the mentioned factor γ , which can enhance the ratio W/W_{trad} , reinforcing the idea that its onerous workload can be decisive to give it up “in favor of more traditional methods” [12].

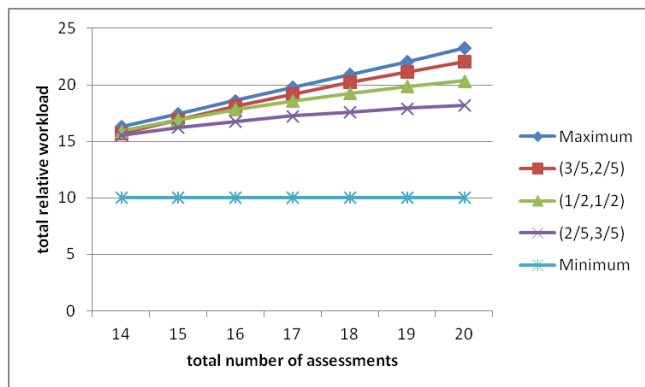


FIG. 6. The total relative workload W/U_1 (vertical axis) versus number of assessments (horizontal axis), for the case with $N_u = 9$ and $N_c = 5$. The behavior of the maximum total relative workload W_{max}/U_1 is given by the diamonds in the blue line, whereas the minimum is given by the horizontal cyan line. The intermediate curves are: for $(\alpha, \beta) = (3/5, 2/5)$, the squares in the red line; for $(\alpha, \beta) = (1/2, 1/2)$, the triangles in the green line; for $(\alpha, \beta) = (2/5, 3/5)$, the crosses in the purple line.

IV. FINAL REMARKS

The mathematical model proposed here, specifically in Sec. II, can describe a very general PSI course. Then, for instance, if one has to investigate the behavior of a course in which to pass an assessment of the 2nd unit is harder than to pass an assessment of the 1st unit, it is possible to set up values of $\beta_{(i,1)}$ and $\beta_{(i,2)}$ so that $\beta_{(i,1)} < \beta_{(i,2)}$ and then build the relations (1)-(3) for this case, which, used in formulas (4)-(7), enable the estimation of the time evolution of the distribution of students per unit of content, the conditions for the inversion of grade, and the amount of workload with assessments. Then, we believe that this general model can be useful for those who are planning or investigating a Keller course. On the other hand, the approximations considered in Sec. III simplifies the visualization of several interesting aspects of PSI courses, as summarized next.

The distribution of students per unit of content propagates as a wave packet, being faster for higher mean values of the β coefficients. The upside down distributions of grade (when the most students achieve the highest grades) observed by Keller, was also predicted by the model discussed here. Furthermore, the present model also shows that this inversion may not occur if the number of assessments is not sufficiently greater than the number of units of content. This information should be taken into account for those who want to plan the appropriate number of assessments of a Keller course, for a given pattern of α and β parameters and defined goals for grades (mastery).

The model discussed here shows that the workload (with assessments) per assessment in the PSI tends to increase until the number of assessments equals to the number of units of content, decreasing next, becoming even smaller than the typical workload per assessment in the traditional scheme. The sum of the workload with each assessment gives the total workload in the PSI, which is usually several times greater than the total workload with assessments in traditional courses. The model enables to estimate this enlargement, reinforcing the idea that the workload requested by the application of the PSI may have been a determinant factor in the decline of its applications.

V. ACKNOWLEDGMENTS

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